Adaptive covariance hybridization for coupled climate reanalysis

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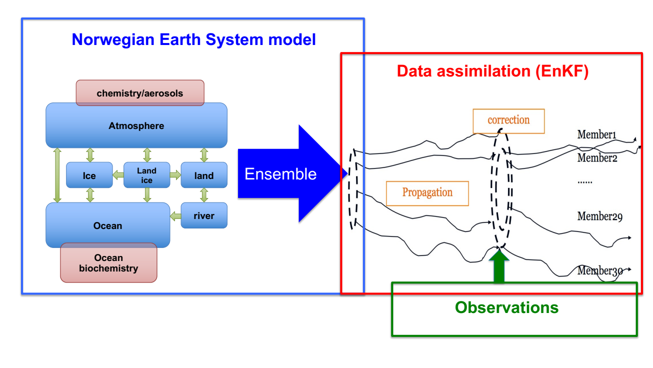
Overview

The Norwegian Climate Prediction Model – NorCPM

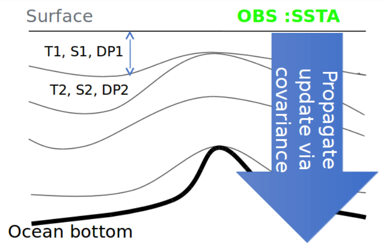
- NorCPM is the combination of the NorESM and the EnKF



Objectives:

- Long climate reanalysis
- Seasonal-to-decadal climate predictions

Data assimilation in NorCPM



- We use **dynamical covariance**
- Covariances are constructed in **isopycnal coordinates**

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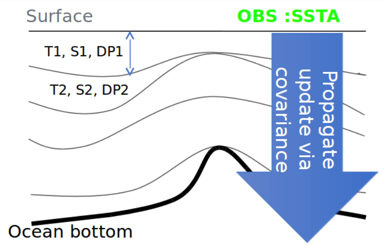

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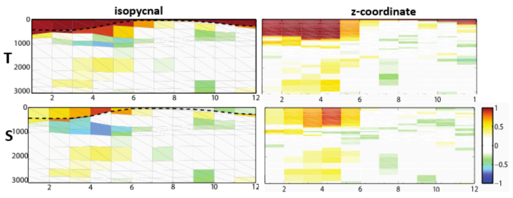
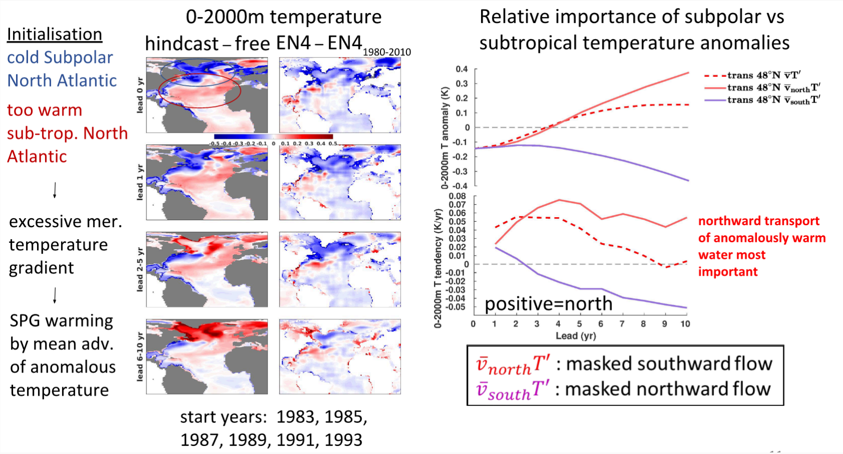
Seasonal correlation of SST in 2010 in the Labrador Sea



- Sharper correlation
- Deeper signature
- Conjugate update of T and S to preserve density

Source: [Counillon *et al.*, 2016]

Composite anomaly patterns of 0-2000 m temperature and salinity



Source [Bethke *et al.*, 2018]

- This problem is due to sampling noise despite computing the covariances in isopycanl coordinates
- What are the possible solutions to address this issue?

Overview

Covariance hybridization – Hamill & Snyder, 2000

The hybrid covariance $\mathsf{P}_h^{\mathrm{f}}$ is a linear combination between:

- a **dynamic** covariance $\mathsf{P}_d^{\mathrm{f}}$ computed from the ensemble (flow dependent but large sampling error).
- a **static** covariance $\mathsf{P}_s^{\mathrm{f}}$ computed from a long stable climatological pre-industrial run (static but lower sampling error).

$$\mathsf{P}_h^{\mathrm{f}} = \alpha_d \mathsf{P}_d^{\mathrm{f}} + \alpha_s \mathsf{P}_s^{\mathrm{f}}, \qquad \alpha_d, \alpha_s \geq 0 \tag{1}$$

- $(\alpha_d, \alpha_s) = (1, 0) \rightarrow$ *full dynamic* case $\approx$ EnKF
- $(\alpha_d, \alpha_s) = (0, 1) \rightarrow$ *full static* case $\approx$ set of EnOI
- Important to tune α_d and α_s to optimal performance:
  - **Empirical tuning:** sensitivity analysis $\Rightarrow$ computationally expensive
  - **Adaptive tuning** of the coefficients

Explicit optimality of the hybridization coefficients

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- The optimal hybrid coefficients are defined as those minimizing the function e:

$$e\left(\alpha_d, \alpha_s\right) = \mathbb{E}\left[\left\|\mathsf{P}_h - \mathsf{P}\right\|^2\right] = \mathbb{E}\left[\left\|\alpha_d \mathsf{P}_d + \alpha_s \mathsf{P}_s - \mathsf{P}\right\|^2\right] \tag{2}$$

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- It can be showed that the optimal coefficients are given by:

$$(\alpha_d, \alpha_s) = \left(\frac{\|\mathsf{P}_s\|^2 \, \mathbb{E}\left[\|\mathsf{P}\|^2\right] - \mathbb{E}\left[\mathsf{P}_d \cdot \mathsf{P}_s\right]^2}{\|\mathsf{P}_s\|^2 \, \mathbb{E}\left[\|\mathsf{P}_d\|^2\right] - \mathbb{E}\left[\mathsf{P}_d \cdot \mathsf{P}_s\right]^2}, \frac{\left(\mathbb{E}\left[\|\mathsf{P}_d\|^2\right] - \mathbb{E}\left[\|\mathsf{P}\|^2\right]\right) \mathbb{E}\left[\mathsf{P}_d \cdot \mathsf{P}_s\right]}{\|\mathsf{P}_s\|^2 \, \mathbb{E}\left[\|\mathsf{P}_d\|^2\right] - \mathbb{E}\left[\mathsf{P}_d \cdot \mathsf{P}_s\right]^2} \right) \quad (3)$$

Properties of the optimal hybridization coefficients

The properties highlighted in [Ménétrier and Auligné, 2015] hold here:

1. **Behavior of the hybridization coefficients:** if P_s is multiplied by a factor λ, then α_s is divided by λ, while α_d remains unchanged $\Rightarrow$ no need for tuning P_s.

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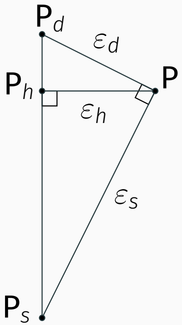

4. **Optimality condition:** P_h verifies the following optimality condition:

$$\left\{\begin{array}{l} \dfrac{\partial e}{\partial \alpha_d} = 0 \\ \dfrac{\partial e}{\partial \alpha_s} = 0 \end{array}\right. \Leftrightarrow \mathbb{E}\left[(\mathsf{P}_d - \mathsf{P}_s) \cdot (\mathsf{P}_h - \mathsf{P})\right] = 0. \quad (4)$$

P_h is the orthognal projection of P on the subspace defined by P_d and P_s

Practical implementation

- (α_d, α_s) can not be computed directly as they are a function of $\mathbb{E}\left[\|\mathsf{P}_d\|^2\right]$, $\mathbb{E}\left[\|\mathsf{P}\|^2\right]$, and $\mathbb{E}\left[\mathsf{P}_d \cdot \mathsf{P}_s\right]$ that are unknown

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- (α_d, α_s) can be estimated using Eqs. (3)-(5) and the **local homogeneity** assumption
- (α_d, α_s) are estimated every $\Delta x = 5$ points and interpolated to the rest of the grid.

Overview

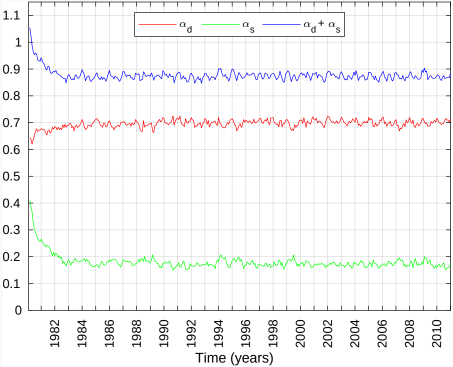
Experimental design

- Monthly assimilation of synthetic SST over 31 years: 1980-2010
- Synthetic SST observations are generated from an independent realisation (TRUE) of the same model with error perturbation matching that of real data (HadISST2)
- **30 dynamic members** and **315 seasonally varying static members** generated from a climatological run with pre-industrial conditions
- 4 different experiments:
  - **FREE:** 30 members run with transient forcing from 1850 to 2014
  - **EnKF:** the standard EnKF used in NorCPM
  - **Standard hybrid:** constant and global hybridization coefficients with $\alpha_d + \alpha_s = 1$. We run 7 versions with $\alpha_d = 0, 0.2, 0.4, 0.6, 0.8, 0.9, 1$
  - **Adaptive hybrid:** the hybridization coefficients are estimated at each assimilation cycle and vary spatially. $\alpha_d + \alpha_s$ can be different from 1

Overview

Variability of the hybridization coefficients with the adaptative method

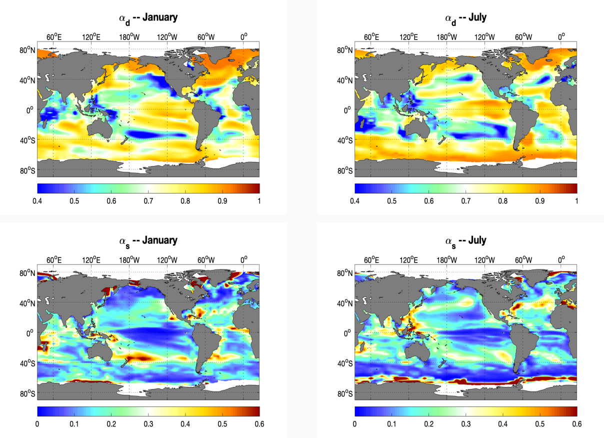
(α_d, α_s) are globally averaged (ice covered regions are masked)



- Convergence of the hybrid coefficients within 3 years
- Some seasonal variability of the coefficients
- $\alpha_d + \alpha_s \leq 1$ (automatic scaling of P_s)

Seasonal variability of (α_d, α_s) with the adaptive method

- ▶ We show the average of the monthly estimates for the period 1983–2010

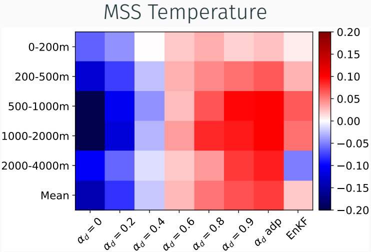


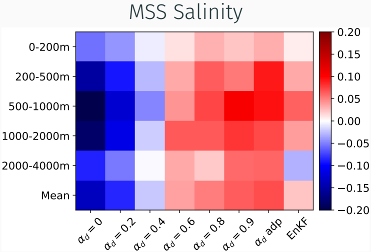
- ▶ α_d and α_s are somehow anti-correlated
- ▶ α_d is large where the internal variability is important, for example in the North Atlantic or the tropical Pacific.
- ▶ Inter-annual deviation from the seasonal estimate is very small (not shown)

Intercomparison of the EnKF and the hybrid covariance schemes

▶ Mean Skill Score (MSS) of one of the nine configurations i: EnKF, adaptive hybrid, standard hybrid with $\alpha_d = 0, 0.1, \ldots, 1$:

$$\text{MSS}_i = 1 - \frac{\text{RMSE}_i}{\frac{1}{9}\sum_{j=1}^{9}\text{RMSE}_j} \tag{6}$$





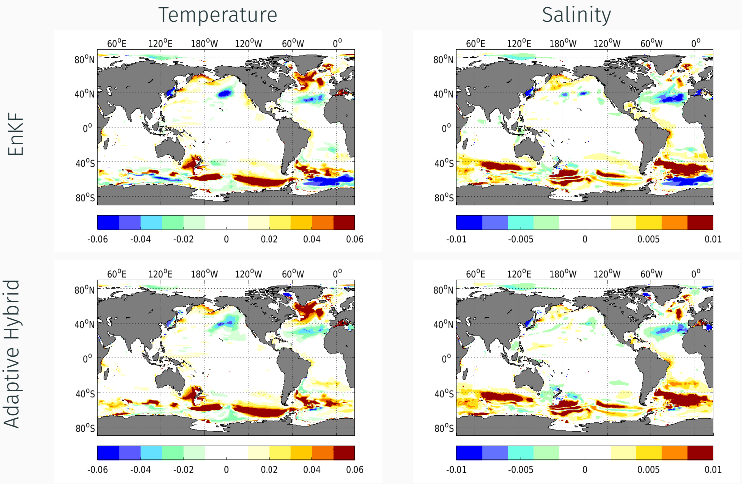
▶ The standard hybrid performs better for large values of $\alpha_d = 0.8, 0.9$

▶ Both the standard hybrid and the adaptive hybrid outperform the EnKF and improve performance substantially between 2000 and 4000m depth

▶ The adaptive hybrid outperforms the standard hybrid

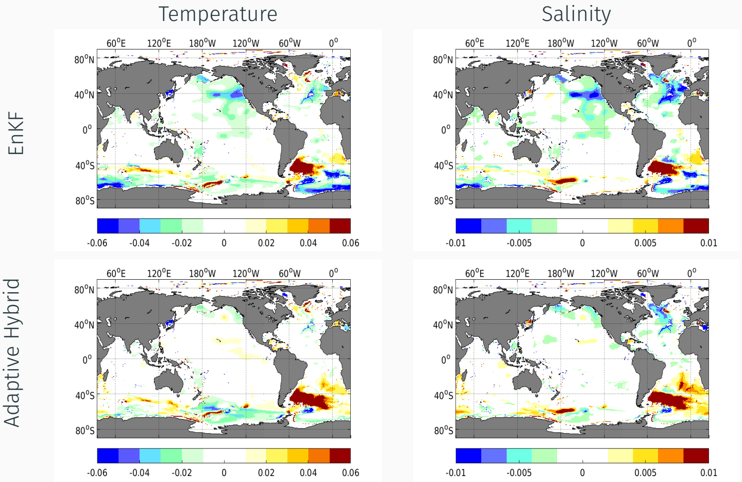
▶ We compare hereafter the adaptive hybrid and the EnKF.

- Difference of pointwise RMSE between FREE and assimilations run (warm colours indicates that assimilation reduces error)



- Improvement in the North Atlantic subpolar gyre
- Mitigate the bias in the north Atlantic and the Southern Ocean.

Difference of RMSE with FREE between 2000–4000m



▶ The adaptive hybrid drastically reduces the degradation seen in the EnKF in the North Pacific and Atlantic, and improves the benefit in the Southern Ocean

Overview

Conclusion

- Development of an adaptive hybrid covariance method (explicit optimality [Ménétrier, 2021]) for the assimilation of SST within NorCPM
- The hybrid covariance schemes outperform the standard EnKF
- The adaptive hybrid outperforms the standard hybrid
- Article in prep. to be submitted to JAMES

Conclusion and perspectives

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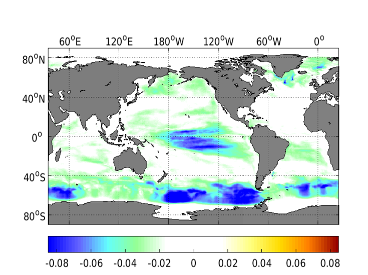
Perspectives

- Testing the method in real framework and with other observations data sets
- Combining with other approaches (isopycnal vertical localisation [Wang *et al.*, 2022])
- It should be used for producing long coupled reanalysis from 1850–present ⇒ project NFR-COREA.

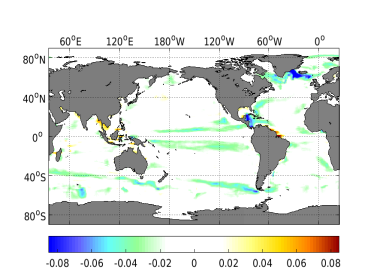
Difference of the degrees of freedom of the signal with the EnKF

▶ DFS $= \mathrm{Tr}\,(\mathbf{KH}) \Rightarrow$ can be interpreted as the amount of observation extracted from the observations. [Cardinali *et al.*, 2004].

EnKF–Standard Hybrid



EnKF–Adaptive Hybrid



▶ The standard Hybrid causes larger assimilation update than the EnKF
▶ The Adaptive Hybrid achieve better performance with nearly similar assimilation updates.